Angular Correlations in the Vector-Meson Exchange Model for Resonance Production*

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The vector-meson exchange model for resonance production is used to predict the angular distribution of the decay products of the resonance and of the scattered particle or resonance. Only parity and angular-momentum conservation rules are used in these calculations. The angular correlation functions are given in explicit form. The examples presented are for reactions of type: $\pi+N\rightarrow\omega+N^*$, $\rho+N^*$, and $\pi+N^*$.

VECTOR-MESON exchange is a possible mechanism for the production of resonances. This model has been considered by Stodolsky and Sakurai[1] who used field theory to obtain the angular distribution of the decay products of the resonance for reactions of the type:

$$\pi^++p\rightarrow N_{3/2}{}^{++}+\pi^0 \qquad N_{3/2}{}^{++}\rightarrow\pi^++p\,, \tag{1}$$

$$K^-+p\rightarrow Y_1{}^{**}+\pi^- \qquad Y_1{}^{**}\rightarrow\Lambda+\pi^+. \tag{2}$$

We consider here the mechanism given in Fig. 1. In all cases, the particle [1] has spin and parity 0(−), particle [3] is the exchange vector meson 1(−), and the particle [4] is a nucleon $\frac{1}{2}(+)$. The particles [6] and [7] are 0(−) and $\frac{1}{2}(+)$, respectively; and the spin and parity of [2] are 0(−) or 1(−). In addition to these particles, it can be significant to measure the angular distribution of the decay products of [2] in correlation with the decay products of [5]; several possible decay modes of [2] will be considered. No other restrictions are imposed on the system. The possible spins and parities are summarized as:

[1]: 0(−)
[2]: 0(−) or 1(−)
[3]: 1(−)
[4]: $\frac{1}{2}(+)$
[5]: a fermion of arbitrary spin and parity
[6]: 0(−)
[7]: $\frac{1}{2}(+)$.

Instead of using field theory, the angular correlation function can be obtained by using only the angular-momentum and parity conservation rules.

The analysis is done in three steps:

(a) The angular momentum state of [3] in the rest frame of [1] is determined from parity and angular momentum conservation rules at the vertex (1,2,3), and by the measurements which can be done on the decay products of [2].

(b) The spin state of [3] is transformed from the rest frame of [1] to the rest frame of [5] (see Appendix).

(c) The angular momentum state of [5] is then given by parity and angular momentum conservation rules at the vertex (3,4,5). This allows us to find the angular distribution of the decay products of [5].

For the analysis of the angular momentum at the vertex (1,2,3), we chose a system of axis such that [1] is at rest, the z axis is along the direction of the momentum of [3] and the y axis is perpendicular to the reaction plane. After the decay of [1], the spin states of [2] and [3], and their orbital angular momentum couple to a total angular momentum $j_1=0$, to form a negative parity state:

$$\psi(j_2j_3;j_{23}l_1;0)=\sum_{m_2m_3}\chi_{j_2}{}^{m_2}\chi_{j_3}{}^{m_3}Y_{l_1}{}^0(0)\langle j_2m_2j_3m_3|j_2j_3j_{23}m_{23}\rangle\times\langle j_{23}m_{23}l_10|j_{23}l_100\rangle. \tag{3}$$

As the values of j_2 and j_3 are restricted to 0 or 1 only, there is only one value of l_1 which satisfies parity and angular-momentum conservation rules: $l_1=1$. For arbitrary values of j_2 and j_3, the summation in Eq. (3) should include the indices l_1 and j_{23}. We write then (in the following, we will drop over-all constants without notice):

$$\psi(j_2j_3;j_{23}l_1;0)=\sum_{m_2}\chi_{j_2}{}^{m_2}\chi_{j_3}{}^{-m_2}\langle j_2m_2j_3-m_2|j_2j_3j_{23}=10\rangle. \tag{4}$$

The observation of the decay products of the particle [2] will select some m_2 states, or some linear combination of m_2 states. The correct combinations will depend on the type of decay of [2], and on the type of measurement made on the decay products. The different cases of interest will be given explicitly later; but in order to keep the formulas simple, we consider now only one specific m_2 state.

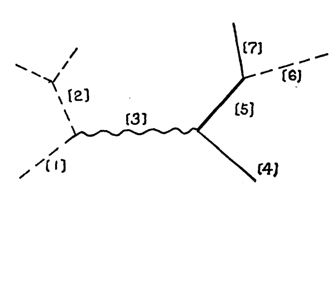

FIG. 1. Schematic diagram of vector-meson exchange model. The particles designations are described in the text.

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[1] L. Stodolsky and J. Sakurai, Phys. Rev. Letters **11**, 90 (1963).

In order to find the spin state of [3] in the rest frame of [5], the following transformations are done:

(1) a Lorentz transformation along the z axis, which brings [3] at rest;
(2) a "rotation" β around the y axis such that the new z axis points in the negative direction of the momentum of [5];
(3) a second Lorentz translation along the new z direction to the rest frame of [5].

If the helicity representation is used,[2] the two Lorentz transformations do not change the spin states. The mixing of states occurs only when the rotation in the rest frame of [3] is performed.

Although the rotation β involves an unphysical angle, it is shown in the Appendix that the transformation matrix for the spin states of [3] looks like the usual $d^{(1)}$ functions. We denote by $U_{j_3}{}^{\mu\nu}(\beta)$ this transformation matrix:

$$\chi_{j_3}{}^{m_3}=\sum_{m_3'} U_{j_3}{}^{m_3'm_3}(\beta)\chi_{j_3}{}^{m_3'}. \tag{5}$$

The spin state of [3] in the rest frame of [5] is then:

$$\psi_{(3)}=\sum_{m_3}\langle j_2m_2j_3-m_2|j_2j_310\rangle U_{j_3}{}^{m_3-m_2}(\beta)\chi_{j_3}{}^{m_3}. \tag{6}$$

At the vertex (3,4,5), the intrinsic spins and orbital angular momentum couple to form a state

$$\psi=\psi_{(3)}Y_{l_2}{}^0\chi_{j_4}{}^{m_4}, \tag{7}$$

or

$$\psi=\sum_{\substack{m_3,l_2\\ j,J}} a_{m_3}\langle j_3m_3l_20|j_3l_2jm_3\rangle \times\langle jm_3j_4m_4|jj_4JM\rangle\chi_J{}^M(j,l_2), \tag{8}$$

where a_{m_3} is the coefficient of $\chi_{j_3}{}^{m_3}$ in Eq. (6). If we denote by H the Hamiltonian which describes the formation of the resonance [5] from the state ψ, the spin state of [5] is

$$\psi_{(5)}=\sum_{m_5}\langle\chi_{j_5}{}^{m_5}|H|\psi\rangle. \tag{9}$$

By using the Wigner-Eckart theorem, this can be written

$$\psi_{(5)}=\sum_{\substack{m_3\\ jl_2}} a_{m_3}\langle j_3m_3l_20|j_3l_2jm\rangle\langle jm_3j_4m_4|jj_4j_5m_5\rangle \times\langle j_5\|H\|jl_2j_5\rangle\chi_{j_5}{}^{m_5}. \tag{10}$$

For any given parity and spin of [5] (with $j_5>\frac{1}{2}$), the conservation of angular momentum and parity restricts the summation over j and l_2 to three terms. They can be expressed in terms of a linear combination of magnetic, electric, and longitudinal multipole wave functions for the vector meson. For example, if [5] is a $\frac{3}{2}(+)$ resonance, we have the possible cases

(1) $j=1;\ l_2=1$
(2) $j=2;\ l_2=1$
(3) $j=2;\ l_2=3.$

The first case corresponds to a $M1$ radiation, and the two last cases to orthogonal mixtures of $E2$ and $L2$. Instead of assuming that one of these three cases is a dominant process, we will keep the summation unrestricted in order to evaluate the interference terms.

The reduced matrix elements $\langle j_5\|H\|jl_2j_5\rangle$ are independent complex numbers.

The particle [5] decays by emitting a $\frac{1}{2}(+)$ [or a $0(-)$] particle in the direction θ, ϕ. A given m_5 state decays to a specific m_7 state as:

$$\chi_{j_5}{}^{m_5}\longrightarrow\langle l_3\mu_3j_7m_7|l_3j_7j_5m_5\rangle Y_{l_3}{}^{\mu_3}(\theta,\phi)\chi_{j_7}{}^{m_7}. \tag{11}$$

In Eq. (11), the summation over l_3 is not indicated, because parity and angular-momentum conservation rules allow only one value of l_3.

For a specific value of m_2, m_4, and m_7, the spatial wave function of [7] is

$$\psi_{(7)}=\sum_{\substack{m_3\\ jl_2}} b\begin{pmatrix} j & l_2 & l_3 & \\ m_2 & m_3 & m_4 & m_7\end{pmatrix}Y_{l_3}{}^{m_3+m_4-m_7}(\theta,\phi), \tag{12}$$

where

$$b\begin{pmatrix} j & l_2 & l_3 & \\ m_2 & m_3 & m_4 & m_7\end{pmatrix} \equiv\langle j_2m_2j_3-m_2|j_2j_310\rangle\langle j_3m_3l_20|j_3l_2jm_3\rangle \times\langle jm_3j_4m_4|jj_4j_5m_3+m_4\rangle \times\langle l_3m_3+m_4-m_7j_7m_7|l_3j_7j_5m_3+m_4\rangle \times U_{j_3}{}^{m_3-m_2}(\beta)\langle j_5\|H\|jl_2j_5\rangle. \tag{13}$$

The angular distribution of [7] is then

$$\omega(\theta,\phi)=\sum\epsilon_{m_2m_2'}b\begin{pmatrix} j & l_2 & l_3 & \\ m_2 & m_3 & m_4 & m_7\end{pmatrix}Y_{l_3}{}^{m_3+m_4-m_7}(\theta,\phi) \times\left[b\begin{pmatrix} j' & l_2' & l_3 & \\ m_2' & m_3' & m_4 & m_7\end{pmatrix}Y_{l_3}{}^{m_3'+m_4-m_7}\right]^*, \tag{14}$$

where the summation is extended over the indices m_2, m_2', m_3, m_3', m_4, m_6, j, j', l_2, and l_2'. Here again, θ, ϕ are the angles of decay of the resonance [5]. The coordinate system is in the rest frame of [5]: $-\mathbf{p}_4$ is the z axis and y is normal to the production plane.

The matrix $\epsilon_{m_2m_2'}$ is the "efficiency matrix" for the observation of the particle [2]. As it is possible to perform several different measurements on [2], we can imagine that different detectors are used, each detector being specified by its efficiency matrix. For example, if only the direction of [2] is measured, no information is

[2] M. Jacob and G. C. Wick, Ann. Phys. (N. Y.) 7, 404 (1959).

obtained on its polarization, and in this case, the efficiency matrix is

$$\epsilon_{m_2 m_2'}=\delta_{m_2 m_2'}\,. \qquad (15)$$

In the case where [2] decays into two π mesons, as can happen if [2] is a ρ meson, a measurement on the directions of the decay products of [2] corresponds to a polarization-direction measurement of [2]. In this case, the efficiency matrix is

$$\epsilon_{m_2 m_2'}=Y_{j_2}{}^{m_2}(\Theta,\Phi)Y_{j_2}{}^{*m_2'}(\Theta,\Phi)\,, \qquad (16)$$

where Θ and Φ are the polar and azimuthal angles of the direction of one of the two π mesons in the rest frame of [2]. This rest frame is related to the rest frame of [1] specified above by a Lorentz translation in the z direction. The coordinates are thus $\mathbf{p}_\pi$ is the z axis and y is the normal to the production plane. If [2] decays into three π mesons, as can be the case if [2] is an ω meson, then

$$\epsilon_{m_2 m_2'}=Y_{j_2}{}^{m_2}(\Theta,\Phi)Y_{j_2}{}^{*m_2'}(\Theta,\Phi)\sin^2(L\psi)\,. \qquad (17)$$

Here, if $\hat{p}_1$ and $\hat{p}_{23}$ specify the directions of propagation of one decay product and the center of mass of the other two, respectively, in the rest frame of [2] (which is again related to the rest frame of [1] specified above by a Lorentz translation in z direction), the angles Θ, Φ specify the direction of the normal to the decay plane: $\hat{p}_1\times\hat{p}_{23}$ and ψ is defined by $\cos\psi=\hat{p}_1\cdot\hat{p}_{23}$. L specifies the relative state that the decay product 1 and the center of mass of the other two are in.

An explicit calculation of (14) for the process

$$\pi^+ + n \to \omega + N^{*+} \qquad (18)$$

through the exchange of a vector meson is carried out and the angular distribution is

$$\begin{aligned}
&\sin^2\Theta\,\sin^2(L\psi)\Bigg\{|a_{11}|^2\left[A(\beta,\Phi)\frac{2+3\sin^2\theta}{18}+B(\beta,\phi,\Phi)\frac{\sin^2\theta}{6}\right]\\
&+|a_{21}|^2\left[\frac{A(\beta,\Phi)}{10}(1+\cos^2\theta)-\frac{B(\beta,\phi,\Phi)}{10}\sin^2\theta+\frac{8}{45}C^2(\beta,\Phi)(1+3\cos^2\theta)-\frac{4\sqrt{2}}{15}C(\beta,\Phi)\,\mathrm{Re}D(\beta,\phi,\Phi)\sin\theta\cos\theta\right]\\
&+|a_{23}|^2\left[\frac{A(\beta,\Phi)}{35}(1+\cos^2\theta)-\frac{B(\beta,\phi,\Phi)}{35}\sin^2\theta+\frac{4}{35}C^2(\beta,\Phi)(1+3\cos^2\theta)+\frac{4\sqrt{2}}{35}C(\beta,\Phi)\,\mathrm{Re}D(\beta,\phi,\Phi)\sin\theta\cos\theta\right]\\
&+2\,\mathrm{Re}(a_{11}a_{21}^*)\left[-\frac{A(\beta,\Phi)}{6\sqrt{5}}(1-3\cos^2\theta)+\frac{B(\beta,\phi,\Phi)}{6\sqrt{5}}\sin^2\theta+\frac{2\sqrt{2}}{3\sqrt{5}}C(\beta,\Phi)\,\mathrm{Re}D(\beta,\phi,\Phi)\sin\theta\cos\theta\right]\\
&-2\,\mathrm{Im}(a_{11}a_{21}^*)\left[\frac{2\sqrt{2}}{3\sqrt{5}}C(\beta,\Phi)\,\mathrm{Im}D(\beta,\phi,\Phi)\sin\theta\cos\theta+\frac{2}{3\sqrt{5}}\,\mathrm{Im}D(\beta,\phi,\Phi)\,\mathrm{Re}D(\beta,\phi,\Phi)\sin^2\theta\right]\\
&-2\,\mathrm{Re}(a_{11}a_{23}^*)\left[\frac{A(\beta,\Phi)}{3(2.5.7)^{1/2}}(1-3\cos^2\theta)+\frac{B(\beta,\phi,\Phi)}{3(2.5.7)^{1/2}}\sin^2\theta+\frac{2}{(5.7)^{1/2}}C(\beta,\Phi)\,\mathrm{Re}D(\beta,\phi,\Phi)\sin\theta\cos\theta\right]\\
&-2\,\mathrm{Im}(a_{11}a_{23}^*)\left[-\frac{2}{(5.7)^{1/2}}C(\beta,\Phi)\,\mathrm{Im}D(\beta,\phi,\Phi)\sin\theta\cos\theta+\frac{2\sqrt{2}}{3(5.7)^{1/2}}\,\mathrm{Im}D(\beta,\Phi,\phi)\,\mathrm{Re}D(\beta,\phi,\Phi)\sin^2\theta\right]\\
&-2\,\mathrm{Re}(a_{21}a_{23}^*)\left[\frac{A(\beta,\Phi)}{5(2.7)^{1/2}}(1+\cos^2\theta)-\frac{B(\beta,\phi,\Phi)}{5(2.7)^{1/2}}\sin^2\theta\right.\\
&\qquad\qquad\left.-\frac{4\sqrt{2}}{3.5\sqrt{7}}C^2(\beta,\Phi)(1+3\cos^2\theta)+\frac{2}{3.5\sqrt{7}}C(\beta,\Phi)\,\mathrm{Re}D(\beta,\phi,\Phi)\sin\theta\cos\theta\right]\\
&\qquad\qquad\qquad\qquad-2\,\mathrm{Im}(a_{21}a_{23}^*)\left[\frac{2}{3\sqrt{7}}C(\beta,\Phi)\,\mathrm{Im}D(\beta,\phi,\Phi)\sin\theta\cos\theta\right]\Bigg\}\,, \qquad (19)
\end{aligned}$$

where

$$\begin{aligned}
&a_{jl_2}\equiv\langle j_5\|H\|j_5 j l_2\rangle\,,\\
&A(\beta,\Phi)\equiv|U_1{}^{11}(\beta)|^2+|U_1{}^{1-1}(\beta)|^2+2U_1{}^{1-1}(\beta)U_1{}^{11}(\beta)\cos^2\Phi\,,\\
&B(\beta,\Phi,\phi)\equiv[|U_1{}^{1-1}(\beta)|^2\cos2(\phi+\Phi)+|U_1{}^{11}(\beta)|^2\cos2(\phi-\Phi)+2U_1{}^{11}(\beta)U_1{}^{1-1}(\beta)\cos^2\phi]\,, \qquad (20)\\
&C(\beta,\Phi)\equiv+iU_1{}^{10}(\beta)\sin\Phi\,,^{3}\\
&D(\beta,\phi,\Phi)\equiv U_1{}^{11}(\beta)e^{i(\phi-\Phi)}+U_1{}^{1-1}(\beta)e^{i(\phi+\Phi)}\,.
\end{aligned}$$

As shown in the Appendix, $\sin\beta$ is pure imaginary in our case with factor $+i$, C is real.

The expressions for the angular distributions in the cases of [2] being a $j_2=0$ particle, or a $j_2=1$ particle which decays into two spin-zero mesons, can be deduced from (19) rather easily. The former can be obtained by replacing

$$U_1^{1-1}(\beta)\to U_1^{0-1}(\beta)\,,\quad U_1^{11}(\beta)\to U_1^{01}(\beta)\,,\quad U_1^{10}(\beta)\to U_1^{00}(\beta)$$

and then integrating over Θ, Φ, ψ. The latter is the same as (19) except that the factor $\sin^2(L\psi)$ must be dropped.

Simultaneous measurements on the decay products of [2] in its center-of-mass system in correlation with the decay distribution of [5] in its center-of-mass system might be necessary in order to determine completely the two relative magnitudes and two relative phases which specify the reduced matrix elements. This type of correlation analysis has been neglected in the experiments up to now. It is seen to require no better statistics than the analysis of the decay distributions of the individual resonances separately and it will yield some independent information.

The complicated form of (19) can be approximated if we believe that $|a_{11}|\gg|a_{21}|$, $|a_{23}|$ corresponding to the argument made by Stodolsky and Sakurai[1] that the magnetic dipole term is predominate. In this case, the angular correlation of decay products of [2] and [5] in their respective rest systems enters through the coefficient $B(\beta,\phi,\Phi)$. We thus have

$$\sin^2\Theta\,\sin^2(L\psi)\{(2/9)(1-3\sin^2\theta\sin^2\phi)+[\cos2(\phi+\Phi)+\cos2(\phi-\Phi)]\tfrac{1}{6}\sin^2\theta-\cos2\Phi((2+3\sin^2\theta)/18)\}\quad(21)$$

which immediately reduces to the result obtained in Ref. 1 once the integrations over Θ, Φ, ψ are carried out. Another approximation is also worthwhile mentioning. Numerical calculation of $U_1^{\mu\nu}$ shows that for incident pion laboratory energy of 3 BeV in the process

$$\pi^++n\to\omega+N^{*+}\,,$$

and for $0>t>-m_\rho^2$, we have $-1<U_1^{1-1}/U_1^{11}<-0.9$ and $iU_1^{10}/U_1^{11}=\sqrt{2}(|U_1^{1-1}/U_1^{11}|)^{1/2}$. Therefore, it would be plausible to make the approximation that $|U_1^{1-1}(\beta)|=|U_1^{11}(\beta)|=\sqrt{2}|U_1^{10}(\beta)|$. Thus, Eq. (19) is simplified to a certain extent. The angular correlation between decay products of [2] and [5] in their respective convenient systems is more explicit, without being obscured by the presence of β dependence among terms.

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APPENDIX

In the rest frame of the resonance [5], the four momenta of [1], [3], [4], [5] are:

$$\begin{pmatrix}P_{1x}\\0\\P_{1z}\\E_1\end{pmatrix},\quad\begin{pmatrix}0\\0\\P_{3z}\\M_3-E_4\end{pmatrix},\quad\begin{pmatrix}0\\0\\-P_{3z}\\E_4\end{pmatrix},\quad\text{and}\quad\begin{pmatrix}0\\0\\0\\M_5\end{pmatrix},$$

respectively.

The Lorentz transformation which brings [3] at rest from the rest frame of [5] is:

$$L=\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1+P_{3z}^2/\sqrt{t}(M_5-E_4+\sqrt{t})&-P_3/\sqrt{t}\\0&0&P_3/\sqrt{t}&(M_5-E_4)/\sqrt{t}\end{pmatrix},$$

where

$$t=M_5^2+M_4^2-2E_4M_5\,,$$
$$S=M_1^2+M_4^2+2E_1E_4+2P_{1z}P_{4z}\,.$$

This transformation in the direction of the motion of [3], does not change its spin state. It is an "unphysical" transformation, corresponding to the crossing of the light cone. In this new reference frame, the four vector of [1] will be

$$L\times\begin{pmatrix}P_{1x}\\0\\P_{1z}\\E_1\end{pmatrix}.$$

We now perform a rotation around the y axis, such that the momentum of [1] lies along the z axis. The rotation matrix has the form

$$\begin{pmatrix}\cos\beta&0&-\sin\beta&0\\0&1&0&0\\\sin\beta&0&\cos\beta&0\\0&0&0&1\end{pmatrix},$$

where

$$\cos\beta=-\frac{(M_5^2-M_4^2)(M_1^2-M_2^2)+t(M_1^2+M_2^2+M_4^2+M_5^2)-2st-t^2}{[(t-M_4^2-M_5^2)^2-4M_4^2M_5^2]^{1/2}[(t+M_1^2-M_2^2)^2-4M_1^2t]^{1/2}},$$

and

$$\sin\beta=i(\cos^2\beta-1)^{1/2}.$$

The rotation β is unphysical, in the sense that $\cos\beta>1$ and $\sin\beta$ is pure imaginary. The corresponding transformation of the helicity state

$$\chi_{j_3}{}^{\mu_3}=\sum_{\nu_3}U_{j_3}{}^{\nu_3\mu_3}(\beta)\chi_{j_3}{}^{\nu_3}$$

can be written as

$$U_1=\begin{pmatrix} (1+\cos\beta)/2 & -(\sin\beta)/\sqrt{2} & (1-\cos\beta)/2 \\ (\sin\beta)/\sqrt{2} & \cos\beta & -(\sin\beta)/\sqrt{2} \\ (1-\cos\beta)/2 & (\sin\beta)/\sqrt{2} & (1+\cos\beta)/2 \end{pmatrix}.$$

Another Lorentz transformation in the new z direction to the rest frame of [1] does not change the helicity state.

The justification of the unphysical transformation can be seen in two ways. From dispersion theory, $\cos\beta$ is the same as the cosine of the angle in the t channel. If, on the other hand, we construct the field function of [3] and impose the Lorentz condition even in the case that [3] is virtual, to eliminate the spin-zero component, we obtain the same answer for the transformation between the field functions in two vertices.

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S-Wave Hyperon-Nucleon Interactions and SU_3 Symmetry

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The generalized Pauli principle is combined with the assumption of SU_3 symmetry to yield relations between hyperon-nucleon and nucleon-nucleon scattering amplitudes for the ${}^1S_0$ and ${}^3S_1$ states.

Although many applications of the "eightfold way" version of unitary symmetry[1] have been made to two-body meson baryon reactions, relatively little attention has been paid to baryon-baryon systems.[2] In this note we combine the assumption of SU_3 symmetry with the generalized Pauli principle to deduce relations between hyperon-nucleon and nucleon-nucleon amplitudes.[3] Particular attention is given to those reactions which are most readily accessible to experiments, namely,

$$p+p\rightarrow p+p \quad T(pp) \tag{1a}$$

$$n+p\rightarrow n+p \quad T(nn) \tag{1b}$$

$$\Sigma^++p\rightarrow\Sigma^++p \quad T(\Sigma^+\Sigma^+) \tag{1c}$$

$$\Sigma^-+p\rightarrow\Sigma^-+p \quad T(\Sigma^-\Sigma^-) \tag{1d}$$

$$\Sigma^-+p\rightarrow\Sigma^0+n \quad T(\Sigma^-\Sigma^0) \tag{1e}$$

$$\Sigma^-+p\rightarrow\Lambda+n \quad T(\Sigma^-\Lambda) \tag{1f}$$

$$\Lambda+p\rightarrow\Lambda+p \quad T(\Lambda\Lambda) \tag{1g}$$

$$\Lambda+p\rightarrow\Sigma^++n \quad T(\Lambda\Sigma^+) \tag{1h}$$

$$\Lambda+p\rightarrow\Sigma^0+p \quad T(\Lambda\Sigma^0) \tag{1i}$$

In general, the wave function of two particles, each of which belongs to an octet representation of SU_3, will be a linear combination of irreducible wave functions belonging to the representations [27], [8_s], [8_a], [10], [$\overline{10}$], and [1]. (Note that the symbol [$\overline{10}$] denotes a continuous bar over the "one" and "zero.") The generalized Pauli principle applied to states containing two baryons which belong to the $J^P=1/2^+$ baryon octet, allows a reduction in the number of independent, reduced SU_3 matrix elements needed to describe the reactions of Eq. (1).

The total wave function for two baryons must be antisymmetric under the interchange of all of the coordinates of the two particles. In the two nucleon problem it is customary to split the total wave function into several parts, one describing the isospin and the other the spin-space part. However, if SU_3 invariance is assumed, then the dichotomy is into an SU_3 part and a spin-space part. Correspondingly, when two baryons are in an antisymmetric spin-space state (${}^1S_0$, ${}^3P_{0,1,2}$, $\cdots$ in the notation ${}^{2S+1}L_J$), their SU_3 wave function $[(B_1B_2+B_2B_1)/\sqrt{2}]$ must be symmetric. B_1 and B_2 represent the SU_3 functions for baryons 1 and 2, respectively. Similarly, for symmetric spin-space states (${}^3S_1$, ${}^1P_1$, $\cdots$) the SU_3 wave function $[(B_1B_2-B_2B_1)/\sqrt{2}]$ is antisymmetric. In general, the SU_3 symmetric states belong to the representations [27], [8_s], and [1], although in the reactions of Eq. (1) the singlet [1] state does not occur. The antisymmetric SU_3 states are contained in [8_a], [10], and [$\overline{10}$].

Assuming strict SU_3 invariance of the strong inter-

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